

A Theory of the Behavior of Municipal Governments: The Case of Internalizing Pollution Externalities^{1, 2}

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I. INTRODUCTION

Although Coase (1960) has argued that private arrangements between damaged and damaging parties may emerge to solve externality problems without outside intervention, his arguments are not accepted as answers to the specific problem of pollution by economists who stress the nature of pollution as a negative public good. Even Demsetz, who argues (1967) that society internalizes externalities by the institution of property rights, considers pollution a problem not amenable to such solution; and most writers of more recent vintage concur that it is impossible to decentralize pollution externalities without direct intervention to control market trends. Baumol (1972), Buchanan and Tullock (1975), Peltzman and Tideman (1972), and Tietenberg (1974a, 1974b) all conclude that this intervention should be effected by Pigouvian pollution taxes rather than by regulatory standards; and Peltzman and Tideman and Tietenberg

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² "Comment on Internalizing Pollution Externalities," by J. Vernon Henderson can be found in 6, 405-406.

additionally stress the desirability of these taxes varying with location. Baumol is also concerned with the problem of finding a measurable criterion by which to determine the desired level of pollution control. Discouraged by the attendant conceptual difficulties, he finally suggests simply choosing an arbitrary level for pollution maintenance, clearly at most a Second Best solution to this problem.

Quite recently, Spann (1976) has pursued a different tack and tried to prove that although the type of arrangement envisioned by Coase may not be taking place between private parties, perhaps it *is* occurring between polluting industries and municipal authorities. If so, an additional pollution tax will cause a misallocation of resources rather than a move toward the expected optimum.

It is the purpose of this paper to continue the line of Spann's thinking. By utilizing the results in Hochman (1978b), we prove that in the short run rational municipal governments have a strong incentive to internalize pollution externalities, and that in the long run, with free entries of cities, such internalization is necessary for their survival. Unlike Baumol, we discover a rather readily available *ex ante* criterion by which to gauge their success: changes in property values and unlike Demsetz, we conclude that the institution of property rights works to internalize pollution as well as other externalities. Either pollution taxes or zoning will suffice, and, depending upon the base on which one views the tax as being levied, this tax varies with location, as Peltzman and Tideman, and Tietenberg deem it should, or is invariant in the Pigouvian mode.

In a recent paper Henderson (1977) proposes a complicated method of internalizing pollution by leveling a combination of emission taxes plus a lump sum tax on the industry. We show that a single, simple Pigouvian pollution tax is sufficient to achieve the same purpose. In addition we consider another method of internalizing externalities often used in practice by local authorities, namely zoning regulations, and we show that if the regulations are imposed on locations rather than firms no distortion of resource allocation results out of it as argued by Buchanan and Tullock.

The main point of departure between Henderson and us is when an equilibrium between cities in the whole economy is considered. Due to the narrow scope of his paper, Henderson concludes wrongly that federal government intervention is needed for the purpose of redistribution of pollution tax proceeds. In a general equilibrium model of an economy with cities, Hochman (1978b) proves that when local externalities and public goods are involved, no intervention of federal government is required in collecting and redistributing tax proceeds justly. In that respect only municipal governments' intervention is needed.

In general, then, intervention by authorities at a higher than local

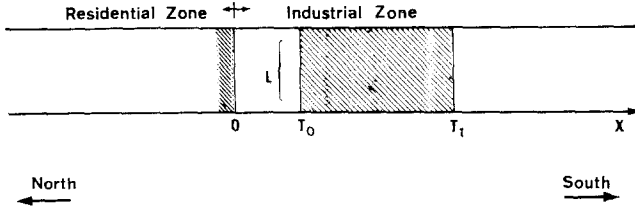


FIGURE 1

level may be not only unnecessary but detrimental to the public. However, there are two possible exceptions: when local response is too slow or when pollution spills across municipal borders. In these events, the proper seat of intervention is that of the authority one step higher in the hierarchical government line, and the proper form of intervention is not direct taxation of the pollution source but rather taxation of or zoning requirements for the government one step below.

II. THE MODEL

We assume the existence of a featureless plain with no amenities and on this plain a linear town with a given width along a given highway. The assumption of a linear town has become standard in recent years (e.g., R. Solow, W. Vickery, and W. J. Stull); the simplification thereby achieved greatly reduces the complexity of the model without affecting the main results.

The town is further considered to be a “small factory town” with a given industry operating in a system of towns. The term “small” simply indicates that the final product of the industry is traded at fixed prices in the rest of the system. In each given location x , the industry is characterized by its production function

$$Z(x) = f[n(x), q(x), La(x)], \tag{1}$$

where

x = distance from the residential area.

$Z(x)$ = output at location x .

$a(x)$ = proportion of the width of the industrial area taken up by the industry at x . L is the total width of the industry strip.

$n(x)dx$ = number of workers employed at x .

$q(x)dx$ = quantity of pollution emitted by the industry at x .

The production function is assumed to be convex and linear homogeneous and to satisfy all the desired neo-classical properties. In particular,

$$\begin{aligned} f_n, f_q, f_a &> 0, \\ f_{nn}, f_{qq}, f_{aa} &< 0. \end{aligned} \tag{2}$$

Neither capital nor economies of scale appear in this world, their presence serving to obfuscate rather than illuminate the analyses. For details on the role of economies of scale in the formation of cities, see Henderson (1974a) and Hochman (1978b). Briefly, they are necessary for the existence of cities of finite size, and a linear homogeneous production function in a featureless plain results in infinitely small cities. We have accordingly cavalierly assumed that cities of finite size exist. Again, the answers to the questions of interest to this study are not fundamentally altered by this simplifying assumption.

We view pollution as a factor of production (or as a negative product) with a negative effect only on the population at its residential location and not in the industry itself. An increase in the level of pollution will increase production, either because cheaper and larger quantities of polluting factors are used (for example, coal instead of oil) or because fewer resources are allocated to pollution abatements. For a similar approach, see Tolley (1974) and Hochman (1978a, 1978b).

It should be further noted that the production function as specified above does not necessarily imply specialization in the town's production activities. Since product prices from the viewpoint of the individual town are fixed, the production function may well be specified in units of value rather than in physical units of the final product. Thus, upon normalization, the final product Z may be regarded as simply composed of homogeneous "pieces of money." This in turn permits production to be heterogeneous: the "industry" may be viewed as producing different physical products with different technologies at different locations.

Turning to the residential-consumption sector, we further assume that the population is homogeneous and can migrate freely between towns in the system. Since the local population is only a small part of the total population in the system, we may assume—following W. J. Stull (1974) and Hochman (1978a)—that the supply is infinite at a given level of utility U_0 . The whole structure of the residential ring is collapsed into a single point which we will indicate as the origin. This enables us to ignore such factors as congestion, transportation cost within the residential area, and the differential impact of pollution on various residential neighborhoods in the town. These aspects, which are important by themselves (see Hochman, 1978a), play only a secondary role in the implications of concern in the present paper. Moreover, including them in the analysis considerably complicates the presentation with only a small gain in generality.

Accordingly, the utility level at the origin is given by:

$$U(C, Q) = U(w + V, Q) = U_0, \quad U_Q < 0, \quad U_C > 0, \quad (3)$$

where Q is the level of concentration of pollution at the origin, and C is

individual consumption. C is determined by the budget constraint, $C = w + V$, where w is the local wage rate and V is non-labor income.

For a discussion of the (long- and short-run) determination of V , see Hochman (1978b) regarding it as exogenous at this stage. The wage rate w , however, is endogenous in the short-run as well as in the long-run. A migrating person (or potential migrant) will be facing different levels of pollution and wages in different cities, in combinations satisfying (3). Note that in previous papers (Henderson, Hochman) the wage rate has been viewed as a utility equilibrating device among cities of different sizes, and it functions now as such a device among cities of different pollution levels.

Addressing the question of pollution accumulation and dispersion, we denote the contribution of the quantity of pollution $q(x)$ discharged at a given distance x from the origin of consumption to the level of pollution at the origin where $x = 0$, by

$$D[q(x), x], \quad D_q > 0, \quad D_x < 0. \quad (4)$$

$D(\cdot)$ is the dispersion function of pollution, indicating that pollution is dispersed and dissipated, diminishing with distance, but varying directly with the amount discharged. The exact nature and shape of $D(\cdot)$ depend of course, on such factors as topography, sunshine intensity, direction and intensity of winds, and temperature, as argued by Tietenberg (1974a, 1974b).

The total accumulated level of pollution at the origin of consumption ($x = 0$) is thus given by

$$Q = \int_{T_0}^{T_1} D[q(x), x] dx, \quad (5)$$

where $x = T_0$ and $x = T_1$ are lower and upper boundaries of the industrial zone.

For further simplicity and no further loss of generality, we may assume $L = 1$. Hence, the constraint of land at each given distance in the industrial zone is given by (6):

$$a(x) \leq 1. \quad (6)$$

Total land used by the industry, assuming a fully exhausted industrial strip, is given by

$$(T_1 - T_0) = \int_{T_0}^{T_1} a(x) dx. \quad (7)$$

Similarly, the size of the local population N , assuming full employment,

is given by

$$N = \int_{T_0}^{T_1} n(x)dx. \quad (8)$$

We are now in a position to introduce the following surplus (or gain) function of the town

$$S = \int_{T_0}^{T_1} [f(n(x), q(x), a(x)) - n(x)(w + t(x)) - a(x)R_A]dx \quad (9)$$

where $t(x)$ is commuting cost per worker between 0 and x , $\partial t(x)/\partial x > 0$, w is the local wage rate (net of commuting cost), and R_A is the agricultural land rent. Hence, S reflects the total value of the industrial output in the town net of the alternative cost of land and the cost of labor.

The maximization of S subject to (3), (5), and (6) is a necessary condition for a Pareto Optimum. If S is not at its optimum value, then by increasing it we can make somebody in the system better off without making anybody else worse off. Similarly, we can argue that a sufficient condition for a Pareto Optimum in the system is that S is at its optimum level in every town in the system. For a detailed proof and further implications, see Hochman (1978a, 1978b).

III. THE SOLUTION OF THE MODEL UNDER OPTIMAL PLANNING

Consider now a planner or city manager with (9) as an objective function to be maximized. The first step to be taken by the planner is to determine an arbitrary line dividing the residential location from the industrial location. It is customary in models of this kind to assume that residents are located on both sides of the industrial zone. However, in dealing with a polluting industry, it seems more reasonable to assume that the industry is located at the outskirts of the town in one particular direction: against the wind. Perhaps that is why in so many industrial towns in the northern hemisphere we find the polluting industries as well as the less attractive residential neighborhoods in the southern end of town, while the more attractive residential neighborhoods are found in their northern parts. We therefore assume that the residential zone is north of the industrial zone as depicted in Fig. 1.

The boundaries of the industrial strip, T_0 and T_1 , are parameters to be determined optimally in the overall solution. The special set-up depicted in Fig. 1 is not meant to rule out the possibility of obtaining $T_0 = 0$, which indicates that the industrial zone is located directly adjacent to the residential zone. (See Appendix for details.)

Another possibility, and one which generally should not be precluded out-of-hand, is a multiple solution for T_0 and T_1 , indicating several industrial zones located at increasing distances from the residential area and separated by agricultural strips. This case is investigated in the Appendix. Because the introduction of such a set-up in our formulation does not affect the final implications in any way, we may however proceed with our analysis on the assumption of a single and continuous industrial zone.

In sum then, the problem of the planner is to maximize (9), subject to (3), (5), and (6) and with respect to the variables under his control: $n(x)$, $q(x)$, $a(x)$, w , Q , T_0 , and T_1 . Let λ_3 , λ_5 , λ_6 denote the respective Lagrange multipliers of (3), (5), and (6). The first order optimality conditions are then:

$$f_n(x) = w + t(x), \quad (10)$$

$$f_a(x) = R_A + \lambda_6(x); \quad \lambda_6(x)(a(x) - 1) = 0, \quad \lambda_6(x) \geq 0, \quad (11)$$

$$f_q(x) = \lambda_5 D_q(x), \quad (12)$$

$$f(T_i) - n(T_i)[w + t(T_i)] - R_A a(T_i) - \lambda_5 D(T_i) = 0 \quad i = 0, 1, \quad (13)$$

$$N + \lambda_3 U_C = 0, \quad (14)$$

$$\lambda_5 - \lambda_3 U_Q = 0. \quad (15)$$

Equations (10) to (15) together with Eqs. (3), (5), and (6) fully determine the optimal solution.

The first order conditions outlined above reflect well-founded economic rationales which characterize the solution. In particular, Eq. (10) indicates that the marginal productivity of labor equals the cost of a unit of labor, and therefore increases with commuting cost as we move farther away from the residential district. Equation (11) indicates that the marginal productivity of industrial land should exceed, or at least not fall short of, the agricultural land rent, since $\lambda_6(x) \geq 0$ everywhere in the industrial district. Because the Lagrange multiplier $\lambda_6(x)$ varies with distance, the marginal productivity of land should vary as well. Note that as long as $f_a \geq R_A$ for $a(x) = 1$ and n^* and q^* , the optimum solution for $a^*(x)$ is unity. However, if for $a(x) = 1$, $f_a < R_A$ then $a^*(x)$ is the solution to $f_a(n^*, q^*, a) = R_A$ which implies $a^*(x) < 1$ and $\lambda_6(x) = 0$.

Solving for λ_5 from (6) and (7), we further obtain

$$\lambda_5 = N(-U_Q/U_C). \quad (16)$$

Thus, the shadow price of a unit of pollution reaching the residential district reflects the value of the marginal damage as assessed by an individual's marginal utility rate of substitution between pollution and

consumption, multiplied by the size of the (happily homogeneous) population residing in the given town. Substituting this shadow price into (12), we get

$$f_q(x) = ND_q(x)(-U_Q/U_C) \quad (17)$$

which means that marginal productivity of a unit of pollution should just offset the marginal damage it is expected to inflict upon the residential district.

The conditions determining the boundaries of the industrial zone are reflected in Eq. (13). This equation may be rewritten, upon substitution from (16), as follows

$$f(T_i) = n(T_i)[w + t(T_i)] + a(T_i)R_A + ND(T_i)(-U_Q/U_C); \quad i = 0. \quad (18)$$

After considering the sufficient conditions (see Appendix) and analyzing the economic rationale behind Eq. (18) we can generalize it as follows:³

$$f(x) \geq n(x)(w + t(x)) + R_A a(x) + ND(x)(-U_Q/U_C) \quad T_0 \leq x \leq T_1. \quad (18')$$

Equation (18') states that the total production at any location should be greater or at least equal to the total social cost caused by production at this location, i.e., total wages paid to the labor, total commuting costs, alternative income from land and the total damages inflicted on society by pollution emitted by industry at this location.

In a similar way we can restate (17) as follows

$$f_q(x) \geq ND_q(-U_Q/U_C). \quad (17')$$

Equality will hold when $D_{qq} \leq 0$. If equality occurs with $D_{qq} > 0$ then we are at a minimum and the optimum at this location is either at $q^* = 0$ or q^* is at a range where $D_{qq} \leq 0$.

Since $f(\cdot)x$ is linear homogeneous we can write, using the necessary conditions

$$f(x) = n(x)(w + t(x)) + f_a a(x) + N_q D_q(x)(-U_Q/U_C). \quad (19)$$

Since at $x = T_i$ there is equality in (18'), by comparing (18) and (19) we get a value for $f_a(T_i)$

$$f_a(T_i) = R_A + \frac{N}{a(T_i)} (-U_Q/U_C)(D - qD_q). \quad (20)$$

Since at $T_i, D_{qq} < 0^4$ then $(D - qD_q) > 0$, hence $f_a(T_i) > R_A$. This,

³ See Appendix Eq. (A15) and arguments leading to it.

⁴ See Appendix for sufficient conditions.

in turn, implies $\lambda_6 > 0$ which implies $a(T_i) = 1$. (18') implies that $f_a(x) > R_A$ for all $T_0 \leq x \leq T_i$. We thus proved that:

$$a(x) = 1 \quad T_0 \leq x \leq T_1. \quad (20')$$

There is a clear analogy between Eqs. (17) and (18'). Both reflect optimal conditions for the utilization of pollution, (17) the "intensive" condition and (18) the "extensive" one. The former is familiar in the literature (see Tietenberg (1974b)). The latter, which refers to optimal location was only recently recognized by Henderson (1977).

Given the optimal solution, the question arises whether it can be achieved by the competitive forces of the free market. This question is taken up in the following sections, first with respect to a differential pollution tax and second with respect to differential zoning regulations. As we shall see, the accepted method of forcing the industry to meet condition (17) in a decentralized market may not insure fulfillment of (18').

IV. INTERNALIZING METHODS

a. Differential Taxation as a Method of Internalization

The customary approach to a pollution problem is to advocate a per unit pollution tax to be imposed upon the industry the cost of the marginal damages of a unit pollution emitted at each location. The marginal damages are given by the right hand side of Eq. (17), and imposing the customary tax will indeed lead to an equality between them and the marginal productivity of pollution. However, all of Eq. (18') may not be fulfilled, that is, the location of the industry may not be optimal.

Henderson (1977) proposed in his paper to impose the marginal damages as tax per unit pollution and then an additional lump sum tax.⁵ His suggestion of an additional lump sum tax would indeed lead to an optimal location of the industry. The method proposed by us, however, is simpler since it requires a single Pigouvian corrective tax.

Accordingly, we argue that the optimal tax to be imposed on $q(x)$, the total pollution discharged at each given location, should rather be derived as follows:

$$PT(x) = N(-U_q/U_c)D[q(x)/a(x), x]a(x), \quad (21)$$

where $PT(x)$ is the pollution tax levied at x . With such a tax and no further intervention, the forces of the free market lead to exactly the same solution optimally effected under planning.

⁵ Note that since D_{qq} can only be negative, only lump sum tax is needed. The case where D_{qq} is positive will never occur, since $q = 0$ is superior to it. Thus no lump subsidy is ever needed.

Under such a system of taxation, the net profit derived at location x from industrial production is given by

$$\Pi(x) = f(x) - n(x)[w + t(x)] - R_t(x) \cdot a(x) - N(-U_Q/U_C)D(x, q(x)/a(x))a(x), \quad (22)$$

where $R_t(x)$ is the rent for industrial land at x . The maximization of Eq. (22), subject to (6), is of course the objective of all industry at location x and yields Eq. (10) and (17) as first-order conditions with respect to $n(x)$ and $q(x)$. These equations together with (3) and (5), which are still valid, yield the optimal solution. We solve for $R_t(x)$ by differentiating (22) with respect to $a(x)$ and then substituting $a(x) = 1^6$ into the result.

$$R_t(x) = f_a(x) + N(-U_Q/U_C)(qD_q(x) - D(x)). \quad (23)$$

It is now easy to see that (23), (10), and (17) imply that in (22) $\Pi(x) = 0$, which means that we are dealing with a competitive industry. Moreover, competition between the industrial and agricultural industries implies that

$$R_t(x) \geq R_A. \quad (12')$$

Since $a(x)$, $n(x)$, and $q(x)$ attain their optimal values for each x , we also have $R_t(T_i) = R_A$. Hence, (12') now implies that the T_i 's are the borders of the industrial zone in this case too. Therefore, the competitive boundaries are identical with the optimal boundaries under planning.

Note that if we regard $q(x)$, the pollution emitted at each plant, as the tax base, then (21) is a differential tax per unit, one which varies with distance as advocated by Pelzman, Tideman, and Tietenberg against the wishes of Baumol and Pigou. If, on the other hand, we consider $D(x)$, the contribution of each plant to the pollution concentration in the origin, as the relevant tax base, (21) is a single tax per unit, the Pigouvian solution.

b. Differential Zoning Regulations as a Method of Internalization

Taxation is not the only method by which internalization of pollution may be achieved: an alternative is differential zoning regulations. This method divides the industrial district, T^*_0 to T^*_1 , into zones, each with a specific maximum level of pollution that may be discharged. At each location x , the maximum amount of pollution a producer is allowed to discharge is given by

$$q(x)/a(x) \leq q^*(x). \quad (24)$$

Because $f_q(x) > 0$, the restriction in (24) will hold as an equality for each

⁶ It can be proven, in a similar way as was proved in the optimal solution that this equality holds for this case as well.

profit-maximizing producer. Thus, from the viewpoint of the producer the restriction is simply

$$q(x) = q^*(x)a(x). \quad (25)$$

The industry in a competitive market under zoning, taking land rents and wage rates as given, will act to maximize $\Pi(x)$ in (26) subject to (6).

$$\Pi(x) = f(n(x), q^*(x)a(x), a(x)) - n(x)[w + t(x)] - R_z(x)a(x), \quad (26)$$

where $R_z(x)$ is the industrial land bid rent under zoning. Note that (3) and (5) imply $w = w^*$ when differentiating with respect to $n(x)$ and substituting $a(x) = a^*(x) = 1$ we get (10) and thus the optimal solution. By differentiating with respect to $a(x)$, we can solve for $R_z(x)$:

$$R_z(x) = f_a(n^*, q^*, 1) + q^*f_q. \quad (27)$$

By then substituting (21) into (27), we obtain

$$R_z(x) = R_t(x) + N(-U_q/U_c)D(x). \quad T_0 \leq x \leq T. \quad (28)$$

Hence, the rent under differential zoning exceeds the rent obtained under taxation by exactly the size of the optimal tax.

The intuitive rationale behind this result is clear. The zoning regulations create pollution rights at the disposal of land owners, who are thereby able to increase their returns by the amount of the pollution tax. These, in turn, equal the value of the total pollution discharged. The producers, in short, now pay their "taxes" in the form of additional rents on land. The important point to observe is that pollution is optimally internalized in both cases, with the only difference being one of income distribution. Note also that this method of differential zoning does not suffer from the faults of regulations as argued by Buchanan and Tullock (1975).

V. THE DECENTRALIZED SOLUTION

In a 1967 paper, Harold Demsetz has argued that there are forces in society that will lead to the internalization of externalities through the establishment of property rights. Yet even Demsetz cannot conceive of a decentralized solution to the problem of pollution externalities. Actually, pollution is only one of many urban externalities, although perhaps somewhat more like a public bad than others, and we argue that society has found a solution to urban externalities by establishing the institution of municipal governments. The role of a municipal government is to provide public goods and services to the industry and population in the city and to internalize urban externalities. In this paper, we are concerned with the second role of local authorities and we are now ready to prove our statements.

Because of our simplifying assumptions, in a competitive system of

cities in short-run equilibrium—that is, with no entry of new cities or industry, the total urban surplus (5) available to municipal governments as “free” income includes only producer and not consumer surplus (cf. Hochman (1978a,b)). Unless each individual in the homogeneous population receives a utility level at least equal to U_0 , he will migrate, thus insuring that his utility be exactly at this level. Following the arguments in Hochman (1978b), a municipal government will act to maximize S_i , using the controls available to it. Those controls being Pigouvian pollution taxes and zoning regulations according to the following rule.

If a government action of taxation or zoning increases the total value of land rents plus government income from taxation, it is a positive one which also increases total welfare, and conversely.

The question of the distribution of municipal income is irrelevant to the efficiency of the system. To deal with it and with the general problems of financing the activities of municipal governments, a broader framework is needed. On one hand equilibrium conditions between cities in the economy has to be derived and other activities of municipal governments, besides internalizing externalities, has to be considered. The interested reader is therefore referred to Hochman (1978b) who deals exactly in those aspects. It is clearly proved there, that no federal government intervention is needed either in internalization externalities or in redistribution of tax proceeds.

In the long run, free entry of cities will drive cities' surplus to zero and drive inefficient cities out of existence as proved in Hochman (b).

Note that at this point Henderson arrived at an opposite conclusion. His arguments lead him to conclude that federal government intervention is required for the purpose of redistribution of income from corrective Pigouvian taxes. His mistake essentially stems from considering a general equilibrium problem in a partial equilibrium framework, such as equilibrium conditions between cities in a model with a single city and without a variety of local governmental activities. Hochman (b) indeed used this wider framework. He derived an optimal method of taxation and distribution of tax proceeds, completely internal to the city and therefore proved that no federal government intervention is needed.

VI. OUT-OF-JURISDICTION EFFECTS AND SHORT RUN-ADJUSTMENTS

The mechanism described so far does not resolve the problem of externalities whose effects are outside of the municipal government's terrain. Nor does it deal with problems of the timing of the approach to long-run optimality.

Because in the first case the local government has no incentive to control

the pollution source and internalize the externality, the intervention of higher authorities is needed. The question then becomes one of which authority and which form of intervention are best. Economists presently propound the use of a pollution tax on point source pollution as the optimal solution to internalizing externalities. However, it is quite apparent from our earlier arguments that this method may cause double taxation by the different authorities and thus a misallocation of resources. Note here that what appear to be property taxes paid to local government may in fact be pollution taxes because coping with externalities by zoning regulations will increase property values, and with them, property taxes. Indeed, Spann's paper shows empirical evidence that this case is quite common.

In the long run, we expect property values and taxes to adjust to the situation of double taxation so that overtaxation will disappear. However, in the short run, there may be severe adjustment problems. Accordingly, our first problem leads right into the second: how to approach long-run equilibrium by a series of optimally timed short-run adjustments, especially because the political balance of power and the irreversibility of most urban investments may make cities too slow in adjusting to changes, considerably delaying the attainment of long-run equilibrium.

These considerations and problems of pollution spilling over political boundaries may require federal or other higher authority action. From the discussion so far, it seems that if the intervention should come in the form of taxation, then each government should tax the government one step down in the hierarchy. This is true both if the pollution damages were outside the jurisdiction of the lower government and if the lower government were not to show a sufficiently speedy response to a problem within its jurisdiction. More concretely, the federal government should limit itself to taxing state governments. The state should tax the county governments, leaving them to tax the cities. Those governments lowest in the hierarchy should tax the actual pollution sources. This framework both avoids double taxation and insures swift responses.

APPENDIX

The formulation of the maximization problem in the text, i.e., max (9) subject to (3), (5), and (6) can be formulated as follows:

$$\max S = \int_{T_0}^T [f(n, q, a) - n(w + t(x)) - aR_A] dx \quad (A1)$$

subject to:

$$U_0 - U(w + v_1Q) \leq 0, \quad (A2)$$

$$\int_{T_0}^{T_1} D(q(x), x)dx - Q \leq 0, \tag{A3}$$

$$a(x) - 1 \leq 0. \tag{A4}$$

Let us define H (the Hamiltonian) as follows

$$H = f(n, q, a) - n(w + t(x)) - aR_A - \delta_2 D(q, x) - \delta_3(x)(a(x) - 1), \tag{A5}$$

and also define L

$$L = \hat{F}(T_0, T_1, w) + \delta_1(U(w + v, Q) - U_0) + \delta_2 Q, \tag{A6}$$

where:

$$\hat{F}(T_0, T_1, w) = \int_{T_0}^{T_1} [f(n(x), q(x), a(x)) - n(x)(w + t(x)) - a(x)R_A + \delta_2 D(q(x), x)]dx \tag{A7}$$

and $n(x), q(x), a(x)$ are given functions and δ_i is a given parameter.

The necessary conditions can now be written as:

$$\partial H / \partial n = \partial H / \partial q = \partial H / \partial a = 0, \tag{A8}$$

$$\partial L / \partial T_0 = \partial L / \partial T_1 = \partial L / \partial W = \partial L / \partial Q = 0. \tag{A9}$$

$$(1) \quad \delta_1 \geq 0 \quad \delta_1(U(w + v, Q) - U_0) = 0,$$

$$(2) \quad \delta_2 \geq 0 \quad \delta_2 \left(Q - \int_{T_0}^{T_1} D(q(x), x)dx \right) = 0,$$

$$(3) \quad \delta_3 \geq 0 \quad (a(x) - 1)\delta_3(x) = 0 \quad \text{for } T_0 \leq x \leq T_1. \tag{A10}$$

Those conditions will also be sufficient if both (I) and (II) are fulfilled: (I) J is concave in $n, q,$ and a for all $T_0 \leq x \leq T_1$ for given $T_0, T_1, w.$ Where J is

$$J = f(n, q, a) - n(w + t(x)) - aR_A \tag{A11}$$

and $D(q, x)$ is convex in q and $(a - 1)$ is convex in $a.$ Thus we assume

(a) $D_{qq} \leq 0$ and (b) $f(\cdot)$ is concave in $n, q,$ and $a.$

$$(I) \text{ is now satisfied. } \tag{A12}$$

(II) $\hat{F}(\cdot)$ is concave in $T_i (i = 0, 1)$ and w and $U(c, Q)$ is concave in c and $Q.$

Let us designate by \hat{F}_i the derivative of \hat{F} with respect to T_i

$$\hat{F}_i = (-1)^{i+1} [f(n(T_i), q(T_i), a(T_i)) - n(T_i)(w + t(T_i)) - a(T_i)R_A + \delta_2 D(q(T_i), T_i)] \tag{A13}$$

then concavity of \hat{F} implies

$$(a) \quad \hat{F}_{ii} = \partial^2 \hat{F} / \partial T_i^2 = (-1)^{i+1} (\delta_2 \dot{D} - n(T_i) i(D)) \leq 0 \quad (A14)$$

where $\delta_2 = -N(U_Q/U_C)$ is calculated from the necessary conditions and dot designate derivation with respect to distance

$$(b) \quad \partial^2 \hat{F} / \partial w^2 = \partial N / \partial w \leq 0$$

$$(c) \quad \begin{vmatrix} \hat{F}_{00} & 0 & n(T_0) \\ 0 & \hat{F}_{11} & -n(T_1) \\ n(T_0) & -n(T_1) & \partial N / \partial w \end{vmatrix} \leq 0$$

$$(d) \quad \hat{F}_{ii} \partial N / \partial W + (-1)^{i+1} n(T_i)^2 \geq 0.$$

If (A14) (a) holds then (A14) (b), (c), and (d) follow from (A12), and the necessary conditions.

Let us thus concentrate on (A14) (a) $-(U_Q/U_C)\dot{D}$ and ni are both positive, so that in general their difference can be both positive and negative, i.e., alternate in sign. Thus if $\hat{F}_1 = 0$ only in one location x_0 and there $\hat{F}_{11} < 0$, then $T_0 = 0$ (a corner solution) and $T_1 = x_0$ and \hat{F}_{11} can have any value at $x = 0$. If, however, at x_0 we have $\hat{F}_{00} < 0$ then $T_0 = x_0$ and $T_1 = +\infty$.

If we have $\hat{F}_i = 0$ at x_0 and x_1 and $\hat{F}_{00}(x_0) < 0$ and $\hat{F}_{11}(x_1) < 0$ then $T_0 = x_0$, $T_1 = x_1$. However, if $\hat{F}_{00}(x_0) > 0$ and $\hat{F}_{11}(x_1) > 0$ then $T_{01} = 0$ (corner solution) $T_{11} = x_0$, $T_{02} = x_1$, $T_{12} = \infty$ (corner solution). The last is an example of a multiple solution where we have two disconnected industrial strips. The economics behind those results provide us with a key to the general case.

$\hat{F}_i(x) = f(x) - n(x)(w + t(x)) - R_A a(x) - \delta_2 D(q(x), x)$ is actually the net social gain at location x . Condition (A14) (a) actually insures that at every location belonging to the optimal solution the net social gain is non negative. We therefore can generalize and substitute equations (A14) (a) and (18) by (A15).

$$f(x) - n(x)(w + t(x)) - R_A a(x) - N \frac{-U_Q}{U_C} D(q(x), x) \geq 0. \quad (A15)$$

i.e. we will produce only at locations where non-negative social gain exist. And as just shown it can result in a disconnected industrial strip depending on relative change of dispersion and commuting costs. In more technical terms it depends on the number of solutions to $\hat{F}_i = 0$. Since the dispersion function depend on the topography of the area and atmospheric conditions it may vary unmonotonically with distance. This may cause multiple solutions to $\hat{F}_i = 0$.

In that respect our model fits a terrain which can certainly be non-featureless. We can substitute (17) and (A12) (a) with the more general (A16), which allows for corner solutions.

$$f_q(x) - ND_q(x)(-U_q/U_c) \geq 0. \quad (\text{A16})$$

Equality will hold if $D_{qq} \leq 0$. If $D_{qq} \geq 0$ it is a minimum and the maximum is either at $q = 0$ or at a range where $D_{qq} \leq 0$.

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